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TECHNICAL REPORT NO. 2

FATIGUE FAILURE CRITERIA FOR COMBINED CYCLIC STRESS

BY

ZVI HASHIN

New 411117

DEPARTMENT OF MATERIALS SCIENCE AND ENGINEERING

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FOR COMBINED CYCLIC STRESS

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Zvi Hashin*

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Failure criteria for combined cyclic stress are represented in terms of parametric families of failure surfaces in stress space. Quadratic approximations and symmetry arguments are employed in systematic fashion to construct isotropic failure criteria for general three dimensional states of cyclic stress. Particular attention has been directed to the important cases of normal stress-shear stress (bending-torsion) and biaxial stress cyclings. It is shown that failure criteria for cycling with and without mean stress (reversed cycling) have different forms, the latter admitting simpler representations.

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1. INTRODUCTION

The present work is concerned with a problem of long standing in fatigue failure research -- the establishment of criteria of failure when the state of cyclic stress is two or three dimensional.

The point of view adopted in the present and most other investigations is that there exists a failure criterion in terms of the cyclic stress components. Thus the aspects of micro-damage or even macro-crack propagation are not considered. The problem may then be stated as follows: Given some simple fatigue failure information such as S-N curves for single stress components, obtained experimentally, construct fatigue failure criteria for a more complicated state of stress.

The problem in this sense resembles establishment of static failure criteria or plastic yielding criteria. There is however an important difference which does not appear to have been emphasized in the literature in the present context. In contrast to static stress a stress cycle is defined in terms of two stresses: maximum and minimum amplitude or equivalently, mean and alternating stress. Thus a failure criterion for combined stress should include both parts for each stress component. This will be discussed later.

Much of previous work on the subject is based on postulates that failure will occur when a certain physical quantity reaches an ultimate value. The quantities used are: principal normal stress, principal normal strain, principal shear stress, complete strain energy density, distortional strain energy density (equivalent to octahedral shear stress criterion). These postulates are well known for static loadings. In cyclic loading the ultimate quantity (e.g., maximum stress) is to be obtained from the appropriate S-N curve.

A well known quadratic criterion for uniaxial stress-shear (bending-torsion) is due to Gough and Pollard [3]. In contrast to previously mentioned criteria it involves two parameters to be determined by experiment.

More recent reviews of various criteria and comparison with test data may be found in [1, 2]. Extensive critical work on evaluation and verification of various criteria is due to Findley et al in numerous papers.

Another type of phenomenological failure criteria is concerned with planes of failure. Ref. [2] is concerned with fatigue failure on the maximum shear plane and the establishment of a general three dimensional failure criterion on this basis. In other work the orientation of the plane of failure may also be an unknown of the problem. See [1, 2] for discussion of such theories.

All of the work mentioned, as well as the present one, are of deterministic nature, thus disregarding the considerable scatter of fatigue test data. There arises the fundamental question: which quantity is a deterministic failure criterion for combined stress cycling supposed to predict? Is it the mean lifetime to failure for a number of tests under identical combined stress cycling, or some other statistical quantity? It appears that an answer to this question has not been given. This problem will not be considered here but its presence and significance should not be ignored.

2. FAILURE CRITERIA

The maximum and minimum amplitudes of a cyclic stress component are denoted σ^2_{ij} , σ^1_{ij} respectively. Then the mean and alternating stresses, σ^m_{ij} and σ^a_{ij}, are defined by

$$\sigma^m_{ij} = \tfrac{1}{2}(\sigma^2_{ij} + \sigma^1_{ij})$$
$$\sigma^a_{ij} = \tfrac{1}{2}(\sigma^2_{ij} - \sigma^1_{ij}) \tag{2.1}$$

It will be assumed that all stresses cycle at same frequency and that their amplitudes do not change during the cycling.

A failure criterion for a three dimensional state of stress determines the number of cycles to failure N for the given state of stress. It will be a function not only of the stress components (2.1) but also of their phase lags δ_{ij}, fig. 1, since it is to be expected that in-phase cycles of stress components will produce different damage than cycles which are not in phase. In general, therefore, the failure condition is

$$F(\sigma^m_{ij}, \sigma^a_{ij}, \delta_{ij}, N) = 1 \tag{2.2}$$

It may be noted in passing that the problem may be further complicated by allowing stress cycle amplitudes to change during cycling. An important special case is application of one stress cycle component first and then another. This situation would require cumulative damage theory under combined stress and will not be considered here.

Phase lags are encountered when the stresses are due to a number of cyclic forces which are not in phase. The writer is not aware of consideration in the literature of their influence on failure under combined stress. It will be henceforth assumed that all stresses cycle in phase and thus the δ_{ij} vanish. Thus (2.1) becomes

$$F(\sigma^m_{ij}, \sigma^a_{ij}, N) = 1 \tag{2.3}$$

It should be noted that sign reversal of any σ^a_{ij} produces a cycling which is half a cycle out of phase with respect to other cycles. Thus this specific out of phase cycling is included in the formulation.

Two important special cases are vanishing alternating stress and vanishing mean stress. In the first case

$$F(\sigma^m_{ij}, 0, N) = F(\sigma^m_{ij}) = 1 \tag{2.4}$$

which is the static failure criterion, while in the second

$$F(0, \sigma^a_{ij}, N) = 1 \tag{2.5}$$

which is a failure criterion for reversed cycling.

The situation is now further simplified by assuming that the ratio

$$R = \frac{\sigma^1_{ij}}{\sigma^2_{ij}} \tag{2.6}$$

is the same for all stress component cycles. This is the case when the body remains elastic under cycling since then all stresses vary linearly with the instantaneous values of the applied cyclic forces. In view of (2.1) and (2.6), (2.3) can be written as

$$F[\tfrac{1}{2}\, \sigma^2_{ij}(1 + R),\ \tfrac{1}{2}\, \sigma^2_{ij}(1 - R),\ N] = 1 \tag{2.7}$$

which will from now on be written

$$F(\sigma_{ij}, R, N) = 1 \tag{2.8}$$

where it is understood that σ_{ij} represent the maximum amplitudes of stress cycles, and that (2.8) is dependent on the value of R. Therefore, any single component S-N curve to be used in obtaining information about the failure criterion must have this same R.

It appears that all previous work on the subject is based on the form (2.8) thus tacitly incorporating the assumptions which have been pointed out above. Consideration of failure criteria of type (2.3) would introduce tremendous additional complexity.

It is conceptually helpful to adopt the usual stress space representation of failure criteria. In such a description (2.8) for a fixed N is a surface in six dimensional stress space (or in three dimensional principal stress space). The surface is the locus of all cyclic stress states (with same frequency and same R ratio) which produce failure after N cycles. When N varies (2.8) becomes a parametric family of surfaces with parameter N. The static failure criterion is defined by the surface N=0. Such an approach has been previously used in [4] for fatigue failure of unidirectional fiber composites in plane stress.

A state of stress σ_{ij} which produces failure after N cycles may be regarded as a vector in stress space connecting the origin to the appropriate point on the N failure surface. It is to be expected on physical grounds that if all σ_{ij} are increased in fixed mutual ratios the number of cycles to failure will decrease. It follows that the failure surface for N_2 is contained within the failure surface for $N_1 < N_2$. Consequently (2.8) is a non-intersecting family of surfaces which are all contained within the static failure surface, fig. 2. (It should be noted that the foregoing reasoning disregards scatter.)

Since infinite failure stresses do not occur in nature the failure surfaces should be closed. However, an infinite failure stress may at times be a convenient mathematical idealization to express the fact that a failure stress for one situation is larger by an order of magnitude than for another. For example: failure under hydrostatic compression as compared to failure under uniaxial stress. In the Mises representation of distortional energy criterion this leads to a cylindrical surface which extends to infinity in octahedral direction.

At this point some function is chosen to approximate the failure surface in a curve fitting sense. Since experience shows that failure surfaces are generally curved the simplest reasonable approximation is a quadratic polynomial. The coefficients of the polynomial must be obtained from test data for loadings which are realizable in the laboratory.

It should be noted that maximum stress and energy density criteria, used in the past, are also of quadratic nature but by the nature of the assumption the coefficients are all predetermined and the only fitting parameter left is the failure value of the stress or energy. It is therefore not surprising that these criteria do not in general fit the test data with accuracy. See e.g., [1].

For purpose of examination of Gough's criterion the special situation when σ_{11} and σ_{12} are the only nonvanishing stresses will be considered. In that event (2.8) is written as

$$f(\sigma_{11}, \sigma_{12}, R, N) = 1 \qquad (2.9)$$

If the material is isotropic or even if it is only transversely isotropic around the direction of σ_{11} the sign of the shear stress can make no difference in failure. (See fig. 3 for illustration.) Therefore (2.9) must be an even function of σ_{12}.

Next (2.9) is approximated by a quadratic polynomial. Thus

$$A\,\sigma_{11}^2 + B\,\sigma_{11}\sigma_{12} + C\,\sigma_{12}^2 + D\,\sigma_{11} + E\,\sigma_{12} = 1 \qquad (2.10)$$

where the coefficients are functions of N. The coefficients B and E must vanish to make (2.10) even in σ_{12}. Furthermore (2.10) must satisfy the one-dimensional S-N curve information

$$\begin{aligned} \sigma_{11} &= 0 & \sigma_{12} &= \sigma_u(R,\ N) \\ \sigma_{11} &= \sigma_u(R,\ N) & \sigma_{12} &= 0 \end{aligned} \qquad (2.11)$$

Consequently, (2.10) assumes the form

$$\begin{aligned} A\,\sigma_{11}^2 + D\,\sigma_{11} + \left(\frac{\sigma_{12}}{\tau_u}\right)^2 &= 1 \qquad (a) \\ A\,\sigma_u^2 + D\,\sigma_u &= 1 \qquad (b) \end{aligned} \qquad (2.12)$$

Additional information to determine A and D must come from failure data under combined stress. It is emphasized that according to previous discussion σ_{11} and σ_{12} cycles must have the same R ratio.

If the cycling is reversed, $R = -1$, then a change of sign of σ_{11} cannot affect the failure for it merely displaces all σ_{11} cycles by half a cycle, which is equivalent to changing the sign of the shear stress with respect to normal stress cycles. Consequently (2.12) must be insensitive to a change of sign of σ_{11} and therefore $D=0$. Thus (2.12) reduces to the simple form

$$\left(\frac{\sigma_{11}}{\sigma_u}\right)^2 + \left(\frac{\sigma_{12}}{\tau_u}\right)^2 = 1 \qquad (2.13)$$

Equ. (2.13) is the Gough "ellipse quadrant" criterion, [3]. An alternate quadratic criterion due to Gough and Pollard [3] is

$$\left(\frac{\sigma_{11}}{\sigma_u}\right)^2 \left(\frac{\sigma_u}{\tau_u} - 1\right) + \frac{\sigma_{11}}{\sigma_u}\left(2 - \frac{\sigma_u}{\tau_u}\right) + \left(\frac{\sigma_{12}}{\tau_u}\right)^2 = 1 \qquad (2.14)$$

which is called the "ellipse arc" criterion and is seen to be a special case of (2.12). It appears that the special forms of the coefficients in (2.14) were obtained by empirical fitting.

The present interpretation of the Gough criteria implies that:

1) σ_{11} and σ_{12} cycles should have the same R ratio. Otherwise the failure criterion is of type (2.3) and (2.13-14) cannot be expected to be valid.

2) Criterion (2.13) should be in better agreement with reversed cycling than with cycling including mean stress.

Tests reported by Gough, [5], include combined cycling with different and with same R ratios. For the case of identical R ratios the results do not especially favor (2.13) over (2.14) for reversed cycling. It should be noted that failure criteria shown in [5] were fitted to the data while by present interpretation they should be expressed in terms of one dimensional failure stresses. On the other hand data of [6] (76S-T61 Aluminum) for various values of mean stress do indicate best agreement with (2.13) for reversed cycling and worsening agreement with increasing mean stress. It would be worthwhile to conduct a testing program with specific purpose to examine this question.

Gough et al have reported that (2.13) is in better agreement with ductile metals (steel) while (2.14) is in better agreement with brittle ones (cast iron). This has also been supported by test data of Findley, [7], including also Aluminum alloys. However, this appears to be a purely empirical observation.

The general criterion (2.8) is now reconsidered on the basis of material symmetry. The present discussion is limited to the case of isotropic materials. Isotropy implies that the failure is independent of the set of axes to which the stress tensor is referred. It follows that (2.8) can at most be a function of the invariants of the stress tensor. Thus

$$G(I_1, I_2, I_3, R, N) = 1 \tag{2.15}$$

where

$$\begin{aligned} I_1 &= \sigma_{11} + \sigma_{22} + \sigma_{33} && = \sigma_1 + \sigma_2 + \sigma_3 && \text{(a)} \\ I_2 &= \sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{13}^2 && = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 && \text{(b)} \quad (2.16) \\ I_3 &= \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{vmatrix} && = \sigma_1\sigma_2\sigma_3 && \text{(c)} \end{aligned}$$

and σ_1, σ_2, σ_3 are the principal stresses associated with σ_{ij}.

Consider first the situation where one of the principal stresses, σ_3 say, vanishes. It follows from (2.16c) that I_3 vanishes. Hence, (2.15) reduces to

$$G(I_1, I_2, I_3, R, N) = g(I_1, I_2) = 1 \tag{2.17}$$

The functional form (2.17) is easily determined from an experiment such as bending-torsion in which the only nonvanishing stresses are σ_{11}, σ_{12}. It follows from (2.16) that for this state of stress

$$\sigma_{11} = I_1 \qquad \sigma_{12} = \sqrt{-I_2} \tag{2.18}$$

If the functional relation (2.9) is known from experiment it follows at once that (2.17) is defined by

$$g(I_1, I_2) = f(I_1, \sqrt{-I_2}) = 1 \tag{2.19}$$

which is equivalent to

$$f(\sigma_1 + \sigma_2, \sqrt{-\sigma_1\sigma_2}) = 1 \qquad (2.20)$$

It would be of interest to examine experimentally whether torsion-bending data can predict biaxial stressing data or vice versa, as stated by (2.20).

As an example consider the Gough form (2.13). Then (2.20) becomes

$$\left(\frac{\sigma_1 + \sigma_2}{\sigma_u}\right)^2 - \frac{\sigma_1\sigma_2}{\tau_u^{\,2}} = 1 \qquad (2.21)$$

Equs. (2.20-21) provide fatigue failure criteria for biaxial reversed stressing and are thus of particular importance for pressure vessels. Rotvel [2] has conducted an extensive series of biaxial principal stress reversed cycling tests with thin walled specimens under internal pressure and axial force cycling, including principal stresses of equal and unequal signs. He found that (2.21) was in good agreement with the experimental data. However, still better agreement was obtained with an empirical criterion given by Crossland, [8], obtained by testing of torsional cycling combined with static hydrostatic pressure. Because of the static stress component the latter criterion falls within the category of the much more complicated criteria (2.3) and therefore does not apply to the test data of [2], for reversed in phase cycling. It must be concluded that the better agreement is fortuitous, bearing in mind the uncertainties of scatter and anisotropy. Fig. 4 reproduces comparison of the two criteria with non-dimensional test data as given in [1].

In the general three dimensional case (2.8) can be written as

$$F(\sigma_1, \sigma_2, \sigma_3, R, N) = 1 \tag{2.22}$$

which is the equivalent of (2.15). The simplest case is reversed cycling, R=-1, of all principal stresses. In this case it is evident on physical grounds that a change of sign of <u>all</u> stresses cannot affect the failure since it merely displaces all cycles by one half cycle. Therefore

$$F(\sigma_1, \sigma_2, \sigma_3, N) = F(-\sigma_1, -\sigma_2, -\sigma_3, N) \tag{2.23}$$

In view of (2.16) the equivalent of relation (2.23) in (2.15) form is:

$$G(I_1, I_2, I_3, N) = G(-I_1, I_2, -I_3, N) \tag{2.24}$$

The failure criterion is now expressed as a polynomial in stresses. To do this it is convenient to express (2.24) as a polynomial in the invariants. Because of the relation (2.24) terms which change sign with simultaneous sign change of I_1 and I_3 cannot appear. Consequently the polynomial has the form

$$G(I_1, I_2, I_3, N) = A_2I_2 + A_{11}I_1^2 + A_{22}I_2^2 + 2\,A_{13}I_1I_3 + \ldots = 1 \tag{2.25}$$

The terms in (2.25) contain stresses up to power 4. Therefore four independent items of testing information are required to determine the different coefficients. It is therefore much simpler to use a quadratic approximation which implies that

$$G(I_1, I_2, I_3, N) = A_2I_2 + A_{11}I_1^2 = 1 \tag{2.26}$$

The coefficients in (2.26) are easily determined in terms of the simple information (2.11) and the identifications (2.18). It follows that (2.26) assumes the form

$$\left(\frac{I_1}{\sigma_u}\right)^2 - \frac{I_2}{\tau_u^2} = 1 \tag{2.27}$$

This is the most general quadratic approximation to the three dimensional isotropic failure condition for reversed cycling.

If the stress cycles have mean stresses the conditions (2.23-24) need not apply. The quadratic approximation to (2.24) is in that case

$$A_1 I_1 + A_{11} I_1^2 - \frac{I_2}{\tau_u^2} = 1 \tag{2.28}$$

where

$$A_1 \sigma_u + A_{11} \sigma_u^2 = 1$$

and σ_u, τ_u are given by (2.11). Additional information for failure under combined stress cycling is needed to determine A_1 and A_{11}. A quadratic approximation of combined σ_{11}, σ_{12} or σ_1, σ_2 test data can be used to model (2.28).

It is apparent that the present approach is entirely dependent on isotropy of the material up to failure. While such an assumption can be criticized it must be borne in mind that its abandonment would complicate the problem by orders of magnitude. For if anisotropy is to be taken into account then the nature of the developing anisotropy as a function of number of cycles and state of stress must be uncovered. Since this is an enormous undertaking it would seem that from a practical point of view failure criteria would have to be limited to modeling for each state of stress separately. For example Gough's criteria for tension-shear should also apply if the material becomes anisotropic, but the connection between this criterion and those for other states of stress would be lost.

3. CONCLUSION

The problem of establishment of deterministic fatigue failure criteria for three dimensional states of cyclic stress has been considered in systematic fashion in terms of quadratic approximations of the failure surface representations in stress space. The failure surface introduced is the locus of all cyclic stress states with identical fatigue lifetime. Thus the failure surfaces are a parametric family with parameter N the fatigue lifetime.

It has been shown that such a failure surface depends in general on the mean and alternating parts of all the cyclic stress components and on their mutual phase lags. Previous work as well as the present one has dealt with the much simpler case where all cycles are in phase and all R ratios of cyclic stress component are identical.

A systematic method of quadratic approximations has been used to derive the Gough criteria for bending-torsion fatigue and general criteria for three dimensional states of stress. The latter are based on assumption of material isotropy up to failure. Special attention has been directed to reversed cycling (R = -1) since in this case the failure criteria admit special simplification.

It is noted that a necessary, and presently lacking, ingredient of treatments such as the present one is recognition of the statistical scatter of fatigue test result and the proper identification of deterministic predictions in terms of means and/or other moments of the random data.

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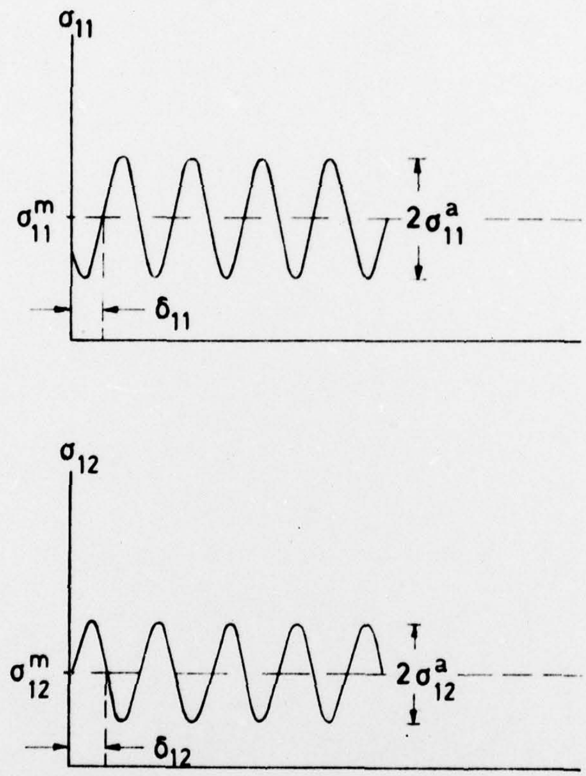

Fig. 1. Combined Stress Cycling with Phase Lags

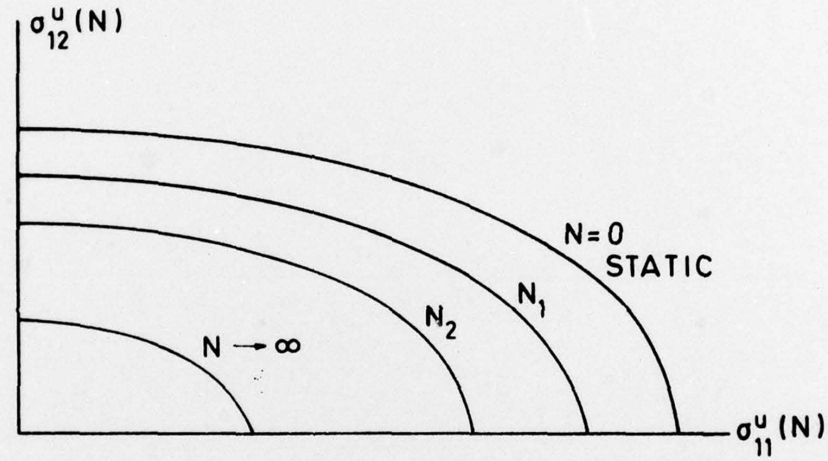

Fig. 2. Failure Surface Family - Schematic

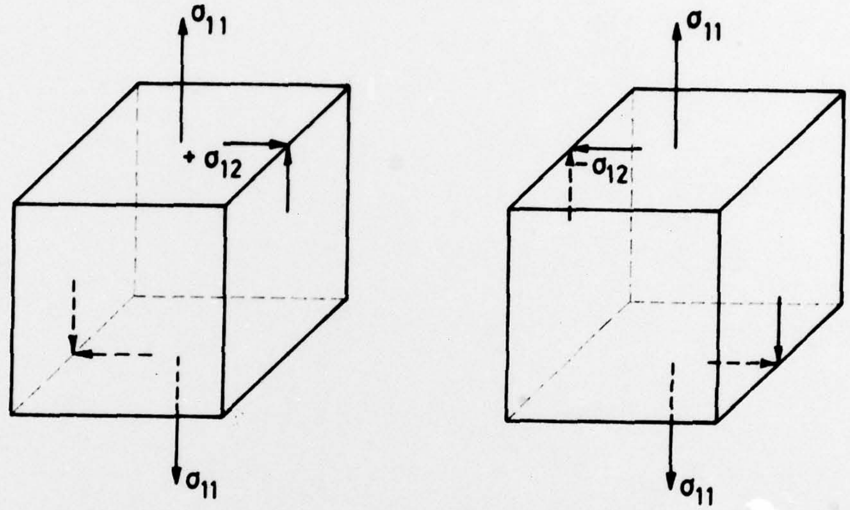

Fig. 3. Sign Reversal of Shear Stress

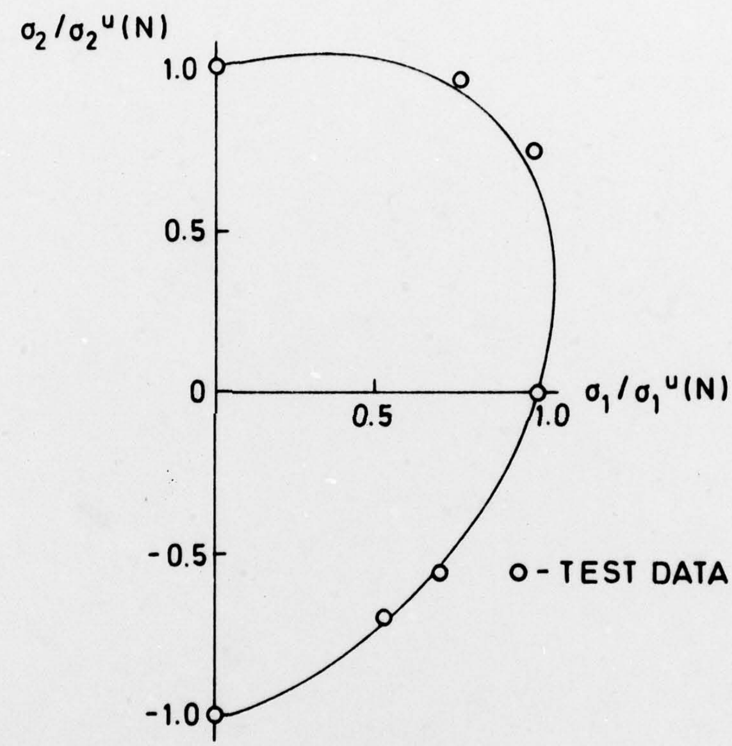

Fig. 4. Comparison of "Ellipse Quadrant" Criterion with Test Data for Reversed Cycling of Thin-Walled Cylinders. (After Rotvel, [2].)